How to Write Mathematics Well

Rules for Content and Presentation

A practical guide for high-school and first-year university students

The central principle

A correct answer is not enough. A mathematical solution must make the reasoning visible: what is being proved, which facts are being used, why each important step is valid, and what conclusion follows.

Contents

1 What is expected in the mathematical reasoning

1.1 Answer the exact question

Identify the task before starting: *compute*, *solve*, *prove*, *justify*, *study*, *deduce*, *give a counterexample*, etc. These verbs do not ask for the same type of response.

Incomplete answer

If the question is "Solve in $\mathbb{R}$: $x^2 - 5x + 6 = 0$", computing the discriminant alone does not answer the question.

A complete answer ends with the solution set:

$$x^2 - 5x + 6 = (x-2)(x-3), \qquad \boxed{S = \{2, 3\}}.$$

1.2 Introduce mathematical objects clearly

A symbol should not appear without being defined by the statement or introduced in the solution. Write, for example:

$$\text{“Let } x \in \mathbb{R}\text{”}, \qquad \text{“For } n \in \mathbb{N},\ u_n = \frac{1}{n+1}\text{”}.$$

Always specify domains when they matter.

1.3 Distinguish calculation from reasoning

A chain of calculations is not always a proof. Use short sentences to explain why the calculation answers the question.

Preferred style

We have

$$x^2 - 4 = (x-2)(x+2).$$

Therefore, by the zero-product property,

$$x = 2 \quad \text{or} \quad x = -2.$$

Hence

$$\boxed{S = \{-2, 2\}}.$$

1.4 Justify important steps

A non-trivial step should be supported by a definition, a theorem, a property, a hypothesis from the statement, or a previous result.

For example, to prove that a sequence is decreasing, do not merely write $u_{n+1} - u_n < 0$; establish the sign and then conclude.

1.5 Check the hypotheses of theorems

A theorem may be used only when its hypotheses are satisfied.

Example: Intermediate Value Theorem

A suitable argument is:

> The function f is continuous on $[a, b]$. Moreover, $f(a) < 0 < f(b)$. Since 0 lies between $f(a)$ and $f(b)$, the Intermediate Value Theorem guarantees the existence of $c \in (a, b)$ such that $f(c) = 0$.

If uniqueness is required, it must be justified separately, for example using strict monotonicity.

1.6 An example is not a proof

Checking several cases can suggest that a statement is true, but it does not prove a universal statement.

To prove that $n^2 + n$ is even for every integer n, write

$$n^2 + n = n(n + 1).$$

Two consecutive integers contain an even integer, so their product is even. Therefore $n^2 + n$ is even for every $n \in \mathbb{Z}$.

1.7 Use implication and equivalence correctly

$$P \Rightarrow Q$$

means "if P, then Q". By contrast,

$$P \Leftrightarrow Q$$

means that both implications are true.

For example,

$$x^2 = 4 \Leftrightarrow x = 2 \text{ or } x = -2.$$

Do not use $\Leftrightarrow$ when only one direction has been established.

1.8 Use the equality sign only for equality

The symbol $=$ does not mean "therefore", "then" or "we obtain".

Incorrect

$$x^2 = 4 = x = 2 \text{ or } x = -2.$$

Correct

$$x^2 = 4,$$

so

$$x = 2 \quad \text{or} \quad x = -2.$$

1.9 Respect domains and conditions of definition

In real analysis:

$$\frac{1}{x-2} \text{ requires } x \neq 2, \qquad \sqrt{x} \text{ requires } x \geq 0, \qquad \ln x \text{ requires } x > 0.$$

Any candidate solution must be checked against the original domain.

1.10 Never divide by a quantity that may be zero without discussion

From

$$x(x-2) = 0,$$

dividing by x would lose the solution $x = 0$. Instead use the zero-product property.

1.11 Conclude explicitly

Do not let a solution stop after a calculation. State what has been proved or found:

$$\boxed{\text{The required length is 3 cm.}}$$

A conclusion should answer the question in its original context.

2 What is expected in the presentation

2.1 Make the structure visible

Number answers clearly: **1(a)**, **1(b)**, **2**, etc. Leave space between different questions. The examiner should never have to guess which question a calculation belongs to.

2.2 Align multi-step calculations

Prefer

$$\begin{aligned} A &= (x+2)^2 - (x-1)^2 \\ &= x^2 + 4x + 4 - (x^2 - 2x + 1) \\ &= 6x + 3. \end{aligned}$$

to a long, crowded line.

2.3 Use sentences, but remain concise

Mathematics is not a sequence of symbols. Sentences are useful to announce a method, cite a theorem, interpret a result and conclude. However, avoid conversational filler such as “Now I am going to try...”.

Prefer: *“Compute $f'(x)$.”* or *“By the Intermediate Value Theorem...”*

2.4 Use notation precisely

Distinguish carefully:

$$f, \qquad f(x), \qquad f', \qquad f'(x).$$

Similarly, (u_n) denotes the sequence, while u_n denotes one term.

2.5 Use exact and approximate values correctly

$$\sqrt{2} \approx 1.414$$

not $\sqrt{2} = 1.414$.

When a physical quantity is involved, include the unit:

$$\boxed{v = 12\ \mathrm{m\,s^{-1}}}.$$

2.6 Highlight final results selectively

Box final answers and important intermediate results, but do not box every line. If everything is emphasized, nothing is emphasized.

2.7 Figures and tables must be usable

A graph or diagram should include the labels, axes, key points and values needed for the reasoning. A sketch may support intuition, but it does not replace a proof unless the question explicitly asks for a graphical reading.

3 Standard forms of mathematical writing

3.1 A proof by induction

A complete induction proof has three clearly identified stages.

Template

Initialization. Verify the statement at the first index.
Induction step. Let $n \in \mathbb{N}$ and assume the statement is true at rank n. Use this assumption to prove it at rank $n+1$.
Conclusion. By mathematical induction, the statement holds for all required n.

3.2 A proof by contradiction

Announce the method. Assume the negation of the desired conclusion, derive a contradiction with a known fact or hypothesis, and then state that the assumption is false.

3.3 A derivative calculation

For example, if

$$f(x) = (x^2+1)e^x,$$

then, since f is a product,

$$\begin{aligned} f'(x) &= 2xe^x + (x^2+1)e^x \\ &= e^x(x^2+2x+1) \\ &= \boxed{e^x(x+1)^2}. \end{aligned}$$

The simplification is useful because it makes the sign of $f'(x)$ immediate.

3.4 A definite integral

A clear solution identifies an antiderivative, evaluates it at the bounds and concludes:

$$\int_0^2 (3x^2+1)\,dx = [x^3+x]_0^2 = 10.$$

Therefore

$$\boxed{\int_0^2 (3x^2+1)\,dx = 10}.$$

4 Finding the right level of detail

Too little detail hides the reasoning; too much detail hides the important ideas. The appropriate rule is:

Rule of thumb

Explain every mathematically meaningful decision and every non-trivial step. Routine operations that are obvious at the expected level may be abbreviated.

For instance, if the question asks you to compute a derivative *from the definition*, writing only $f'(x) = 2x$ is insufficient even if the result is correct.

5 A complete example

Let

$$f(x) = x^2 - 4x + 1.$$

Determine the variations of f on $\mathbb{R}$.

Model answer

The polynomial function f is differentiable on $\mathbb{R}$ and

$$f'(x) = 2x - 4 = 2(x-2).$$

Hence $f'(x) < 0$ for $x < 2$, $f'(2) = 0$, and $f'(x) > 0$ for $x > 2$.
Therefore f is strictly decreasing on $(-\infty, 2]$ and strictly increasing on $[2, +\infty)$. Moreover,

$$f(2) = 4 - 8 + 1 = -3.$$

Thus f has a minimum value

$$\boxed{-3}$$

attained at

$$\boxed{x = 2}.$$

The strength of this answer is not its length: it is the fact that every logical connection is visible.

6 Checklist before handing in your paper

- ☐ I answered every question and clearly numbered my answers.
- ☐ I introduced the variables and objects I used.
- ☐ I stated relevant domains and conditions of definition.
- ☐ I justified every important step.
- ☐ I checked the hypotheses of every theorem I used.
- ☐ I used $=$, $\Rightarrow$ and $\Leftrightarrow$ correctly.
- ☐ I did not divide by a potentially zero expression without discussion.
- ☐ My calculations are aligned and readable.
- ☐ I distinguished exact values from approximations.
- ☐ I included units where required.
- ☐ I clearly stated the final conclusion of each question.
- ☐ My results are plausible and consistent with the problem.
- ☐ Another student could follow my solution without an oral explanation.

Remember

A strong mathematical solution usually follows the pattern

$$\boxed{\text{State} \longrightarrow \text{justify} \longrightarrow \text{calculate} \longrightarrow \text{interpret} \longrightarrow \text{conclude.}}$$

The goal is not merely to show the answer. The goal is to show why the answer is mathematically correct.